A01-532E-Y01

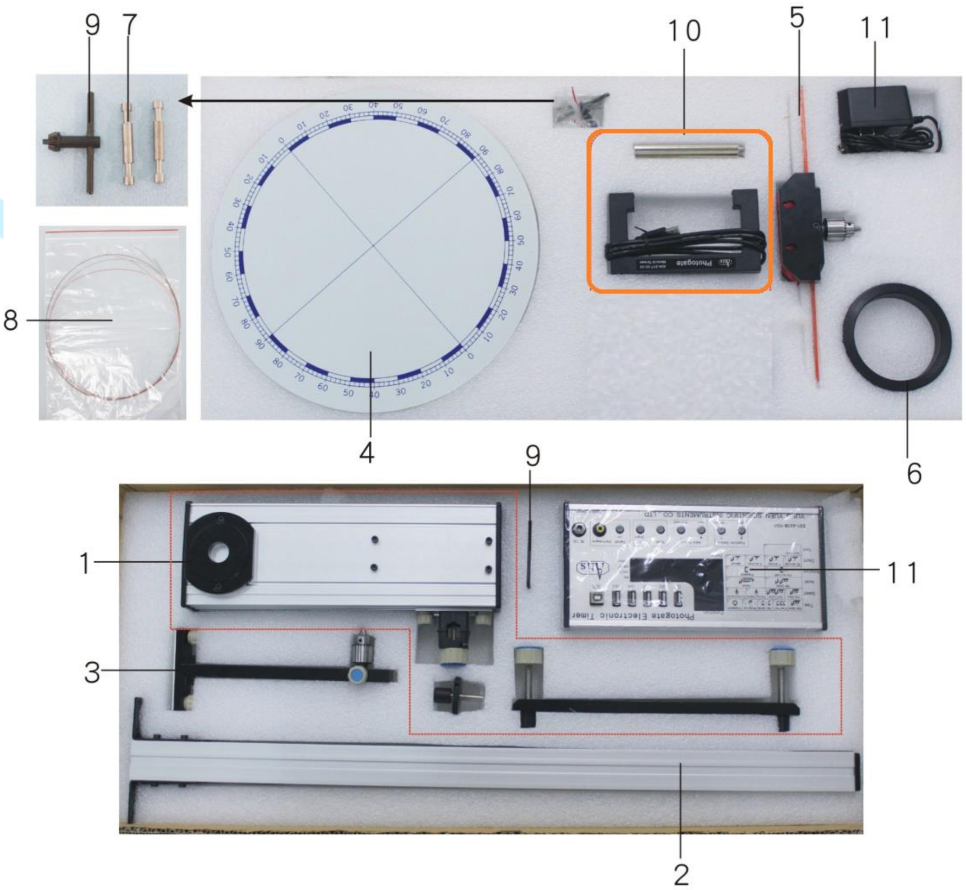
Torsion pendulum experiment

I. Experiment purpose：

By testing torsional period gets the modulus of rigidity of metal wire.

II. Experiment device：

Experiment device list					
No.	Name of device	Qty.	No.	Name of device	Qty.
1	Aluminum alloy base (attached moveable plug, three-point adjustable feet)	1set	2	Aluminum alloy rack	1unit
3	fixed cantilever equipment	1set	4	Angle plate	1unit
5	Ring fixed base	1unit	6	Ring	1unit
7	fixed bolt	2unit	8	Testing wire	3piece
9	Tool (attached L and T shape tool)	2unit	10	Photogate sensor (including iron bar)	optional
11	Dynamic data capture device (attached power supply 12VDC)	optional			



III. Experiment theory：

Any elastomer, within elastic limit, the ratio of **stress** and **strain** is constant e called elastic coefficient. To a rigid body, the ration of shearing stress and shearing strain is n called rigidity coefficient.

As image 3-1, a rectangular rigid body from up and bottom sides have one couple of F force and make body change. If the included angle ϕ is small, then:

shearing stress＝ F/A

shearing strain＝$\overline{HH'}/\overline{HK} = \tan\phi \cong \phi$

n（rigidity coefficient of object）＝shearing stress / shearing strain＝$\dfrac{F/A}{\phi}$ （1）

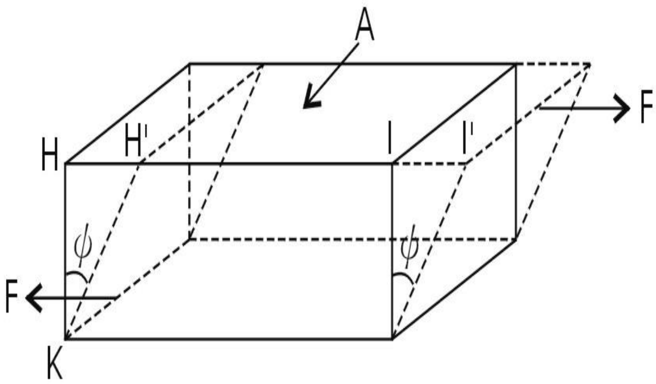


Image 3-1

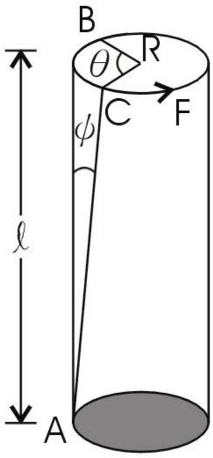


Image 3-2

Image 3-2, solid cylinder length λ, radius is R, $\overline{AB}$ is a vertical line, fixed A point without moving in the bottom and one torque force F are along to the edge from point B to point C. When torque, the central axis without moving, $\overline{AB}$ and $\overline{AC}$ is included angle ϕ, and the central angle of arc BC is θ, so:

BC arc $= R\theta \cong \lambda\phi \ (\lambda >> R)$ （2）

As image 3-3, we select a radius r which width is dr. If the tangential force on the ring is dF, then we can get from formula (1) :

$$n=\frac{F/A}{\phi}=\left(\frac{dF}{dA}\right)\frac{1}{\phi}=\left(\frac{dF}{2\pi\, rdr}\right)\frac{1}{\phi}$$

$$dF=2\pi n\phi\, rdr$$

To entire of solid cylinder, the total torque on the top of surface is :

$$\tau=\int r\, dF$$

$$=\int_0^R 2\pi n\phi r^2 dr$$, we put (2) to this formula and eliminate ϕ

$$=\int_0^R 2\pi n\left(\frac{r\theta}{\lambda}\right) r^2 dr$$

$$=\frac{n\pi\theta R^4}{2\lambda}$$

$$\tau=\frac{n\pi\theta R^4}{2\lambda} \qquad (3)$$

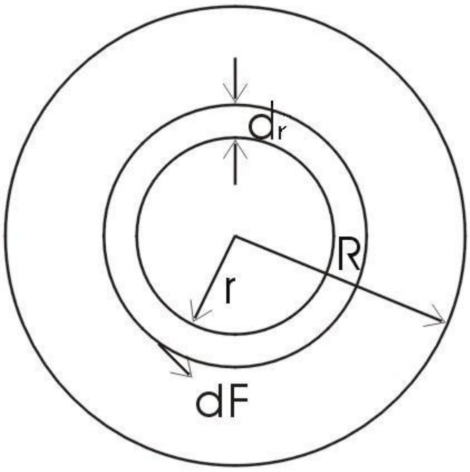


Image 3-3

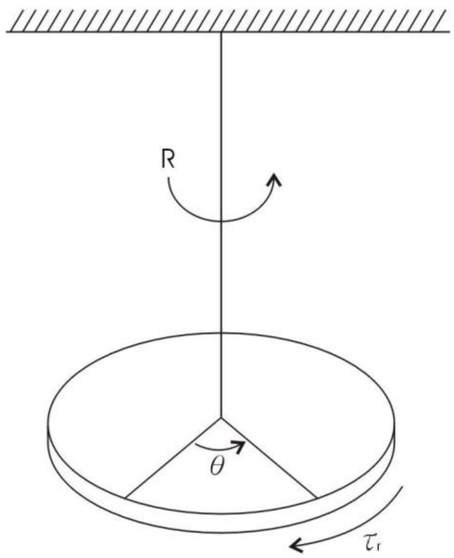


Image 3-4

As image 3-4, we torque angle θ, the wire has a restoring torque, and this is direct ratio to θ.

$$\tau_r=-K\theta$$

K is a constant of torque, and it is a wire feature constant. We can get from (3) formula.

$$\frac{\tau}{\theta}=\frac{n\pi R^4}{2\lambda}=K \qquad (4)$$

Image 3-4 torque has a restoring torque τ_r. We set torsion pendulum of moment of inertia to be I, and :

$$\tau_r = -K\theta = I\alpha = I\ddot{\theta} \quad \left(\ddot{\theta} = \frac{d^2\theta}{dt^2}\right)$$

$$\ddot{\theta} + \frac{K}{I}\theta = 0 \quad \Rightarrow \quad \omega^2 = \frac{K}{I}$$

we can get T (period of torsion pendulum) $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{K}}$ （5）

From (4) and (5), we get torsion pendulum coefficient of rigidity：$n = \frac{8\pi\lambda I}{T^2R^4}$

From this experiment we cannot directly get the moment of inertia I_0 in the base of disc. Thereby, during experiment we need to measure the period of pendulum in horizontal pendulum and vertical pendulum in order to eliminate the moment of inertia I_0 in the base of disc. Then we get K value.

When displaying horizontally ：the total moment of inertia $I_b = I_0 + I_1$，I_1 is a circle and horizontal moment of inertia,

Period of $T_b = 2\pi\sqrt{\frac{I_b}{K}} \Rightarrow K = \frac{4\pi^2(I_0 + I_1)}{T_b^2}$ （6）

When displaying vertically：the total moment of inertia $I_c = I_0 + I_2$，I_2 is a circle and vertical moment of inertia,

Period of $T_c = 2\pi\sqrt{\frac{I_c}{K}} \Rightarrow K = \frac{4\pi^2(I_0 + I_2)}{T_c^2}$ （7）

From (6) and (7) we get

$$K = \frac{4\pi^2(I_0 + I_1)}{T_b^2} - \frac{4\pi^2(I_0 + I_2)}{T_c^2} = 4\pi^2\frac{(I_1 - I_2)}{T_b^2 - T_c^2}$$

bring K value to formula (4) and we get metal wire coefficient of stiffness.

$n = \frac{8\pi\lambda}{R^4}\frac{(I_1 - I_2)}{T_b^2 - T_c^2}$ （8）

As Image 3-5: the horizontal moment of inertia in the disc.

$$I_1 = M\frac{b^2 + c^2}{2}$$

$$I_2 = M(\frac{b^2 + c^2}{4} + \frac{a^2}{12})$$ the vertical moment of inertia in the disc.

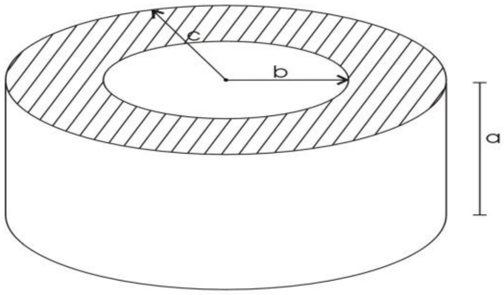


Image 3-5

IV. Experiment step：

1. Test a ring mass M, and measure the thickness of the ring a, inside radius, and outer radius with a vernier scale. We recode the measured numbers and bring them to formula to get I_1 、I_2.
2. A wire stick lock into clamping frame on a wall λ inside of a screw and the other side lock into a clamp on a metal base as image 4-1 and 4-2. We lock up to avoid falling stuffs during this experiment.

Image 4-1

Image 4-2

3. We measure the length of the wire λ by a tape ruler, and measure the radius of the wire by a micrometer caliper. The radius of wire is the fourth power to the coefficient of rigidity. It has tolerance easily. Therefore, we pick up five different places on the wire and measure for five times to get an average value. We record the number in chart 1.
4. We display ring horizontally under metal base as Image 4-3 and attach light mask on metal base as Image 4-4.

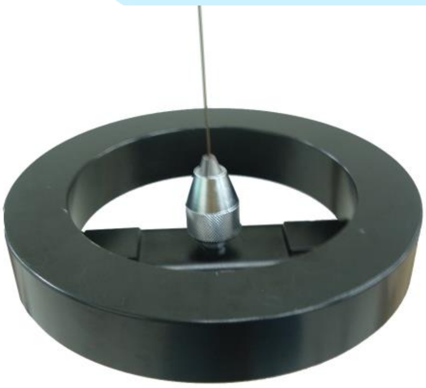


Image 4-3

Image 4-4

5. Install photogate as Image 4-5. We use removeable plug to combine photogate holder and photogate sensor from the base and install a round rod for the photogate. Finally, we connect with round rod and support rack by securing clip.

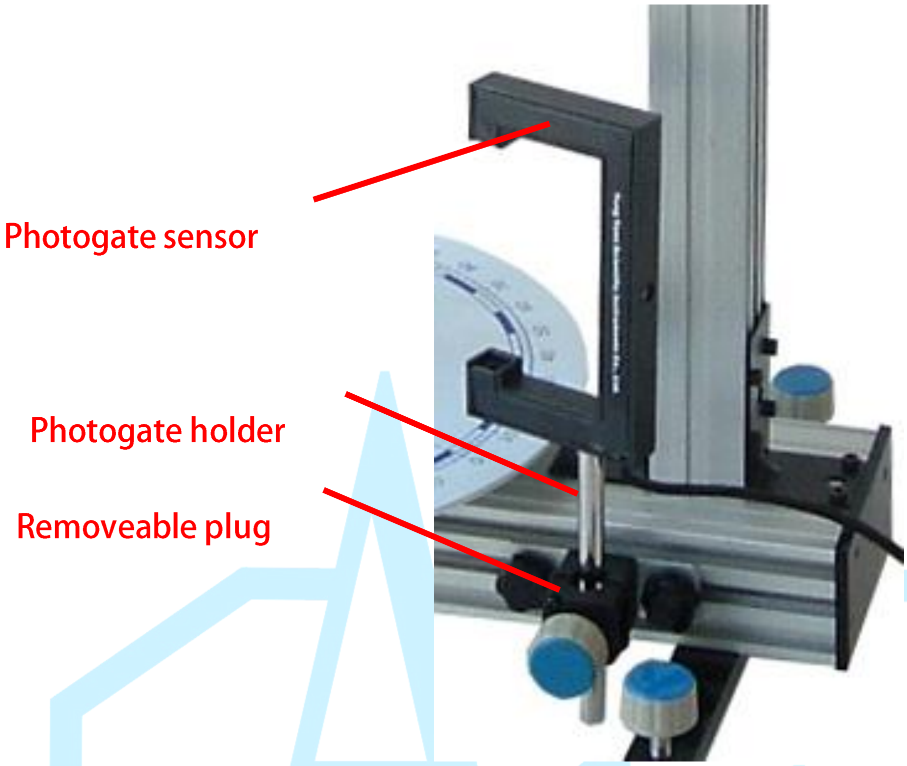


Image 4-5

6. We set a high for photogate to be able to mask the light when torsion pendulum, as image 4-6.

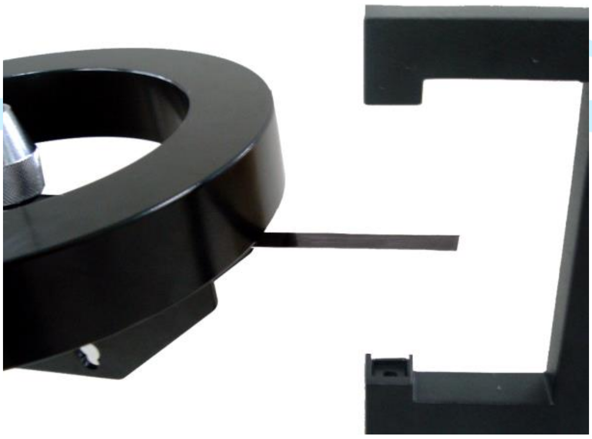


Image 4-6。

7. The photogate UBS connect to Ch1 on dynamic capture device and set Function 4.
8. We turn 60° on the ring. (Try not to swing center axis and the mask piece do not cover the light before releasing the ring.) Then we release slowly to make it turn.
9. After pendulum, the mask sensor will automatically record pendulum period for 10 sets. After 10 sets, the sensor will record and display in order. User can use Date Select button to review history. If we continue to record the next 10 sets period, we can put Reset button.

We can record in chart 2.

10. We hang the ring vertically on metal base, as image 4-7 and 4-8. We repeat step 7~9 and complete chart 2.

Image 4-7

Image 4-8

11. We change a wire to repeat the experiment step and complete chart 3 and 4.
12. We change unknow material metal wire and repeat to finish chart 5 and 6.
13. We change unknow material metal wire and repeat to finish chart 7 and 8.

V. Experiment data：

Ring measurement

Mass：_______kg	Thickness a：_______ m	Horizontal moment of inertia I_1 ：_______$kg \cdot m^2$
inside radius b：_______ m	outer radius c：_______ m	Vertical moment of inertia I_2 ：_______$kg \cdot m^2$

Chart 1： Metal wire information

Material：			Length of wire：_______m			
Time	1	2	3	4	5	Average
Wire Radius R						（m）

Chart 2：Metal wire coefficient of rigidity

Time	display horizontally		display vertically		coefficient of rigidity n（N/m^2）
	20 times in total	Period T_b（s）	20 times in total	Period T_c（s）	
1					
2					
3					
4					
5					

Average $\bar{n}$ ＝__________ （N/m^2）

Chart 3： Metal wire information

Material：			The length of wire：_______m			
Time	1	2	3	4	5	Average
Wire Radius R						（m）

Chart 4：Metal wire coefficient of rigidity

Time	display horizontally		display vertically		coefficient of rigidity n（N/m^2）
	20 times in total	Period T_b（s）	20 times in total	Period T_c（s）	
1					
2					
3					

4					
5					

Average $\bar{n}$ = __________ (N/m^2)

Chart 5：Metal wire information

Material：			The length of wire：_______m			
Time	1	2	3	4	5	Average
Wire radius R						(m)

Chart 6：Metal wire coefficient of rigidity

Time	display horizontally		display vertically		coefficient of rigidity n (N/m^2)
	20 times in total	Period T_b (s)	20 times in total	Period T_c (s)	
1					
2					
3					
4					
5					

Average $\bar{n}$ =___________ (N/m^2)

Chart 7：metal wire information

Material：			The length of wire：_______m			
Time	1	2	3	4	5	Average
Wire radius R						(m)

Chart 8：Metal wire coefficient of rigidity

Time	display horizontally		display vertically		coefficient of rigidity n (N/m^2)
	20 times in total	Period T_b (s)	20 times in total	Period T_c (s)	
1					
2					
3					
4					
5					

Average $\bar{n}$ =___________ (N/m^2)

VI. Question and discussion：

1. How do we test the moment of inertia of relative mass axis of irregular object by this experiment method?
2. What will it happen when turning the angle too much? Will torsion pendulum period be changed because of angle displacement different?
3. Whether torsion pendulum period is the same with different length and the same of material and radius of metal wire? And if the coefficient of rigidity will be different?
4. If this experiment is in moon, will torsion pendulum period be the same and will the coefficient of rigidity be the same?

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