Torsion Apparatus

Purpose

1. Measure the torsion constant of the metal rod.

2. Measure and calculate the modulus of rigidity of the metal rod.

Theory

Any flexible elastic objects, within its' flexibility limit. The ratio of stress and strain is the constant – e. e as the torsion modulus.

For rigid objects, the ratio of shearing stress and shearing strain is n, which is called the modulus of rigidity.

(Figure 2-1) The rectangular rigid object becomes deformed when a couple of forces F were applied to the upper and bottom side. If the angle ϕ is small, one can get:

Shearing stress$=$ F/A

Shearing strain$=$ $\overline{HH'}/\overline{HK} = \tan\phi \cong \phi$

n（Object's modulus of rigidity）$=$Shearing stress / Shearing strain $= \dfrac{F/A}{\phi}$ （1）

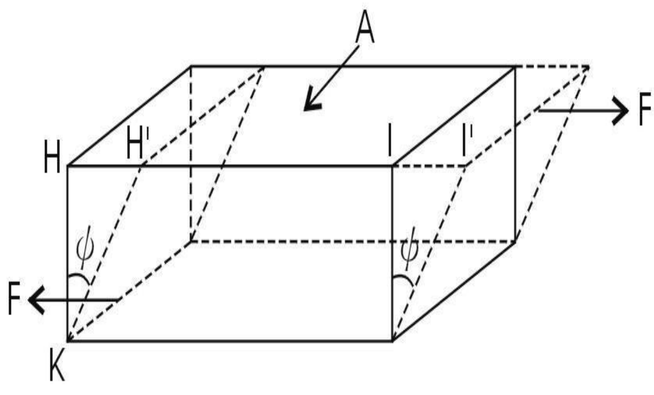


Figure 2-1

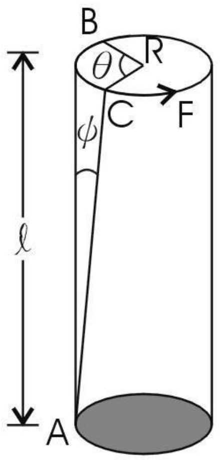


Figure 2-2

(Figure 2-2) Solid cylindrical body length , radius R, $\overline{AB}$ as the vertical line of the side. Keep the bottom A still, one torque force F along with the edge of the upper side, which can make B turns to the C position. During the process, the central axis does not swing. ϕ as the angle in between $\overline{AB}$ and $\overline{AC}$, θ as the angle of the radian BC.

Thus: Radian BC $= R\theta \cong \ \phi$ ($>>$R) （2）

As figure 2-3, the radius on the inner ring on the cylindrical top is “r”. “dr” as the width of the ring. If the force to the ring is dF, we can get below from equation（1）:

$$n = \frac{F/A}{\phi} = \left(\frac{dF}{dA}\right)\frac{1}{\phi} = \left(\frac{dF}{2\pi\, rdr}\right)\frac{1}{\phi}$$

$$dF = 2\pi n\phi\, rdr$$

The total torque to the top of the whole cylinder: $\tau = \int r\, dF$

$$= \int_0^R 2\pi n\phi\, r^2 dr$$ ，Take equation （2）back in and get rid of ϕ

$$= \int_0^R 2\pi n\left(\frac{r\theta}{\ }\right) r^2 dr$$

$$= \frac{n\pi\theta R^4}{2}$$

$$\tau = \frac{n\pi\theta R^4}{2} \qquad (3)$$

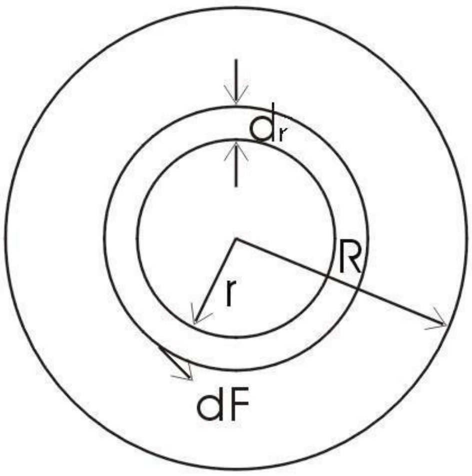


Figure 2-3

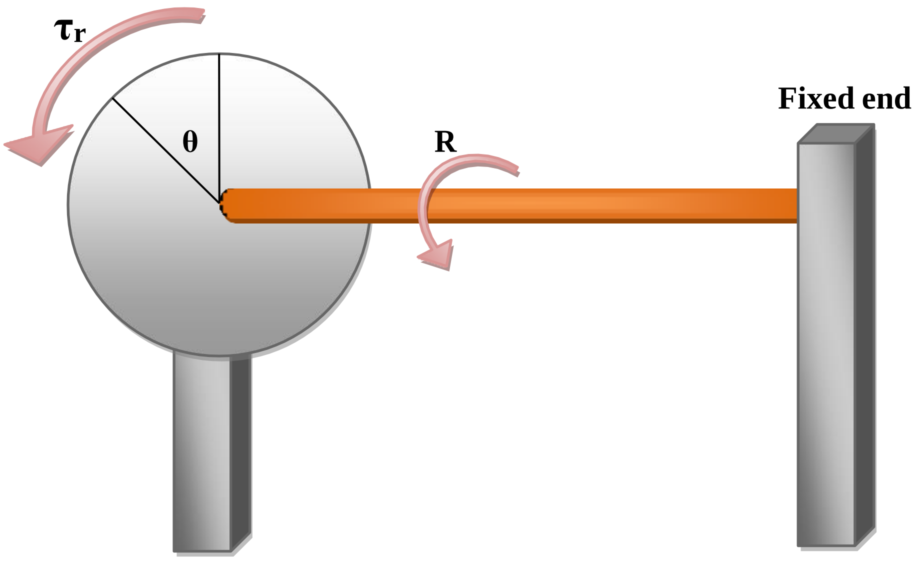


Figure 2-4

As figure 2-4. Apply a force to make the disk turn for θ angle. A recovery torque would appear to become proportional to θ.

$\tau_r = -K\theta$

K is called torsion constant which is the coefficient of the metal rod.

From equation （3）, one can get：

$$\frac{\tau}{\theta} = \frac{n\pi R^4}{2} = K \qquad (4)$$

Figure 2-4 The recovery torque as τ_r, Set the moment inertia as I. Therefore：

$$\tau_r = -K\theta = I\alpha = I\theta \quad \left(\theta = \frac{d^2\theta}{dt^2}\right)$$

$$\theta + \frac{K}{I}\theta = 0 \quad \Rightarrow \quad \omega^2 = \frac{K}{I}$$

One can get T（Torsion periods）$= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{K}}$ （5）

The modulus of rigidity from equation （4）、（5）：$n = \frac{8\pi I}{T^2 R^4}$

By adding the weights to the disk individually for the pendulum periods measurement, which would take away the original moment of inertia of the disk I_0 and to get the value of K.

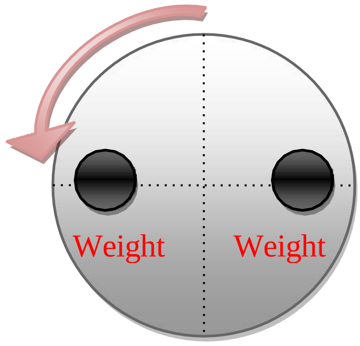


Figure 2-5

Place 100g weight each symmetrically as figure 2-5 above：

Moment of inertia for the whole body $I_b = I_0 + I_1$

I_1 as the moment inertia of the 100g*2 weights added on the disk.

$$\text{Period} \quad T_b = 2\pi\sqrt{\frac{I_b}{K}} \Rightarrow K = \frac{4\pi^2(I_0 + I_1)}{T_b^2} \qquad (6)$$

Place 200g weight each symmetrically：

Moment of inertia for whole body $I_c = I_0 + I_2$

I_2 as the moment inertia of the 200g*2 weights added on the disk.

$$\text{Period} \quad T_c = 2\pi\sqrt{\frac{I_c}{K}} \Rightarrow K = \frac{4\pi^2(I_0 + I_2)}{T_c^2} \qquad (7)$$

From（6）—（7）, one can get:

$$K = \frac{4\pi^2(I_0 + I_1)}{T_b^2} - \frac{4\pi^2(I_0 + I_2)}{T_c^2} = 4\pi^2\frac{(I_1 - I_2)}{T_b^2 - T_c^2}$$

Put the value of K from the above formula back into equation（4）, one can get the modulus of rigidity for metal wire:

$$n = \frac{8\pi}{R^4}\frac{(I_1 - I_2)}{T_b^2 - T_c^2} \qquad (8)$$

(Additional adding) Moment of inertia：

Use the cylindrical coordinates to calculate the moment of inertia of the disc weights as figure 2-6 below, you can get:

$$I_1 = \frac{1}{2} M (b^2 + c^2)$$

(M: Total value / b: Inner diameter / c: Outer diameter)

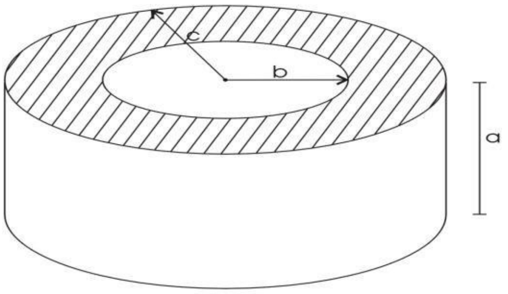
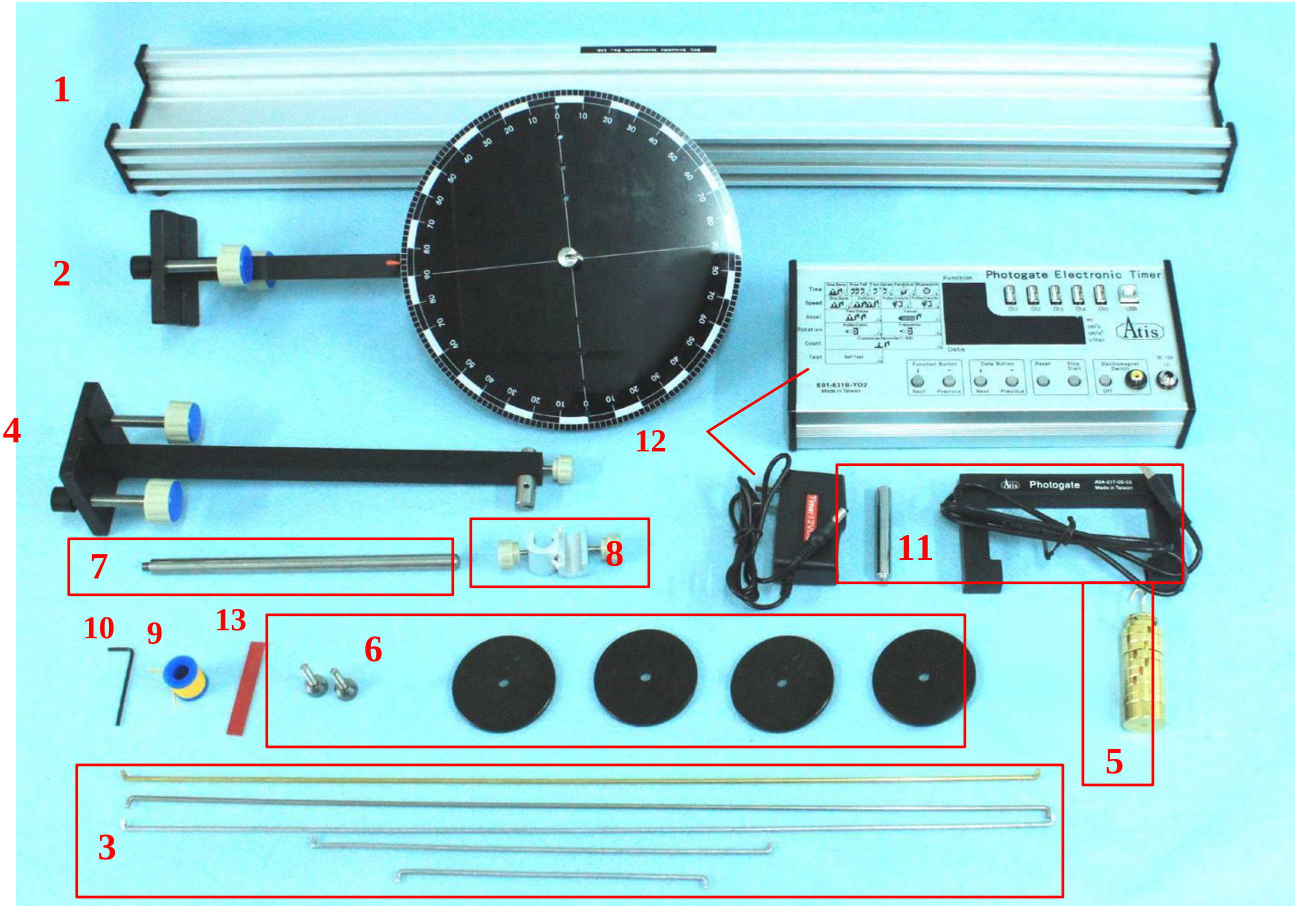


Figure 2-6 Disc weights

Use the parallel axis theorem: $I=I_{CM}+Md^2$ (d: Distance between two axis) to get the inertia of the disc weights placed on round disk.

Instrument



No.	Accessory	Qty	No	Accessory	Qty
1	Experiment base(L80cm)	1	2	Angle disk holder	1
3	Rod to be measured×5 (Please base on the actual goods: steel rodψ3×L530 、cooper rodψ3×L530 、aluminum rodψ3×L530 、ψ3×L26 、ψ3×L170 ，Unit:mm)	1	4	Movable rod holder	1
5	Weight set: Weight holder 50g×1 Weight 20g×10, 10g×1, 5g×2	1	6	Weight set for disk: screw×2 weight with hole 100g×4	1
7	Rod with teeth (H20cm)	1	8	Crossed connector	1
9	Thread	1	10	Hex key	1
A01-532E-Y31 Torsion Apparatus (Photogate electronic)					
11	Photogate sensor (Include iron rod*1)	Optional (1)	12	Photogate electronic timer (Include power supply*1)	Optional (1)
13	Red acrylic sheet	Optional (1)			