

Circular Pendulum

I. Purpose

- 1. Discuss ring pendulum and arc pendulum period.
- 2. Use photogate to record pendulum period and compare to the theory.

II. Theory

Use parallel axis theorem and the symmetry of ring and arc to calculate the period of $T = 2\pi \sqrt{\frac{2r}{g}}$



As image 1, if there is a mass m with radius r of a ring, when the mass center spins, the moment of inertia is $I_C = mr^2$. If we take a P point on the ring, when spinning (OP = d = r), according to the parallel axis theorem the moment of inertia is:

$$I_p = I_C + md^2 = mr^2 + md^2 = 2mr^2$$

as a small angle of pendulum period is:

$$T_{P} = 2\pi \sqrt{\frac{I_{P}}{mgd}} = 2\pi \sqrt{\frac{2mr^{2}}{mgr}} = 2\pi \sqrt{\frac{2r}{g}} = T_{Ring}$$

g: Acceleration of gravity. T_{Ring} : ring of pendulum period.

As image 2, when cut ring pendulum in half, the mass is $\frac{1}{2}$ m, and the mass center location shifts for a distance from the center of ring O. The moment of inertia of mass of arc pendulum is I_{Arc} :

$$I_{Arc} = \frac{1}{2}m(r^2 - a^2)$$

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Image 2

We set a fulcrum P' on the arc pendulum and the moment of inertia is T_{Arc} :

$$T_{Arc} = 2\pi \sqrt{\frac{mr(r-a)}{mg(r-a)}} = 2\pi \sqrt{\frac{2r}{g}}$$

We know the period is the same as the ring period.

$$T_{Arc} = 2\pi \sqrt{\frac{2r}{g}} = T_{Ring}$$



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III. Accessory

(For reference only, subject to the actual sample)

| No. | Name | Qty. |
|----------------------------|---|------------------|
| 1. | H shape aluminum base with adjustable level feet | 1 set |
| 2. | Aluminum frame (H70cm) | 1 set |
| 3. | moveable connector base | 1 unit |
| 4. | steel V shape axis | 1 piece |
| 5. | Ring and arc pendulum with diameter 40cm, 30cm and 20cm. half ring with | 1 set |
| | 40cm diameter, 3/4 ring and 1/4 ring. | |
| 6. | Aluminum plug fixed base | Optional 1unit |
| 7. | Connector rod | Optional 1 piece |
| 8. | Photogate sensor (USB Type A male connector), with 3cm pillar. | Optional 1 piece |
| 9. | Dynamic data capture (with power supply) | Optional 1 set |
| Experiment accessory image | | |
| A 2 7 1 5 | | |

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IV. Experiment

Experiment one: Demonstrate ring and arc pendulum simultaneously.

- 1. According to experiment accessory image, we steady and assemble with knob. We place the device on a table with a proper level by using adjustable feet.
- 2. We set a ring and a half of ring (1/2 arc pendulum) on the frame and ensure that the aluminum ring is matching to knife gap (not just on one point), as image.
- 3. We use a ruler to push a little angle and release. We observe the status of the two pendulums.
- 4. We change different ring and arc pendulum and compare to for discussing the relation.

Experiment two: Use photogate to record the period.

- 5. We install aluminum plug fixed base on aluminum track frame and lower than removeable plug base as upper image.
- 6. There are 3cm pillar in the side of the photogate for connecting to a connector rod that can install on aluminum connector fixed base and with dynamic data capture select Function 4 to record pendulum period.
- 7. We can use opaque tape with length 20mm and width 5mm on ring and arc pendulum to stick on the surface when masking as below image.



8. We record the periods and calculate periods by theory to compare in each.

