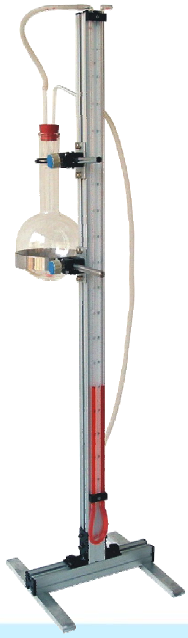
Experiment: Determination of $\gamma \equiv \frac{C_P}{C_V}$



Purpose

Measure the ratio γ of the specific heats of air according to the method of Clement and Desormes.

$$\gamma = \frac{C_P}{C_V}$$

Theory

In thermodynamics, an object's heat capacity (symbol C) is defined as the ratio of the amount of heat energy transferred to an object to the resulting increase in temperature of the object,

$$C = \frac{\Delta Q}{\Delta T}$$

The heat capacity per mole of a gas at constant volume is called molar heat capacity at constant volume C_V; and the molar heat capacity per mole of a gas at constant pressure is called molar heat capacity at constant pressure C_P.

$$c_V = \frac{1}{n}\left(\frac{dQ}{dT}\right)_V \qquad (1)$$

$$c_p = \frac{1}{n}\left(\frac{dQ}{dT}\right)_p \tag{2}$$

By the first law of thermodynamics; the internal energy of a closed system changes as heat and work are transferred in or out of it.

$$dQ = dU + dW \tag{3}$$

So the ideal gas equation can be written as

$$pv = nRT \tag{4}$$

(n is the particle number of moles；

R = 8.31 J/mol-K is the gas constant)

dW in equation (3) can be written as

$$dW = pdv \tag{5}$$

At constant volume, substitute dv=0 into equation (3), so equation (1) can be written as

$$c_V = \left(\frac{dU}{dT}\right)_V$$

$$\Rightarrow dU = c_V dT \tag{6}$$

According to equation (5) and (6), equation (3) can be written as

$$dQ = c_V dT + pdv \tag{7}$$

The differential equation of a mole of ideal gas is

$$vdp + pdv = RdT$$

Substitute the above equation into equation (7)

$$dQ = (R + c_V)dT - vdp$$

At constant pressure, dp=0

$$(\frac{dQ}{dT})_p = R + c_V = c_P \tag{8}$$

Consider in the adiabatic state, dQ=0 , so equation (7) can be expressed as

$$c_V dT + pdv = 0$$

Substitute the deal gas equation into the above equation and do the integration, then the substitute the result into equation (8)

$$\frac{dT}{T} + \frac{R}{c_V}\frac{dv}{v} = 0$$

$$\Rightarrow const = Tv^{\frac{R}{c_V}}$$

$$\Rightarrow const = \frac{p}{R}v^{\frac{c_V + R}{c_V}}$$

$$(c_V + R = c_P, \ \gamma = \frac{c_P}{c_V})$$

$$\Rightarrow const = pv^{\gamma} \tag{9}$$

So $\gamma = \frac{c_P}{c_V}$ is the heat capacity ratio.

According to thermodynamics look-up tables, we know the values Cv and Cp of the ideal gas.

$$c_V = \tfrac{5}{2}R$$
$$c_P = \tfrac{7}{2}R$$

$$\Rightarrow \gamma_{air} \cong 1.4$$

Use a U-shaped tube of water as a water column manometer for measuring pressure. First, set the pressure in the experimental container given by the air ball as *Po* and the room temperature as *Tr*. While stable, open the switch gate quickly to release the gas. When the pressure in the container reaches the room pressure *Pr,* close the switch gate quickly. At this time, the air volume in the container changes from *Vo* to *V1* and the temperature changes from *Tr* to *T1*. Because the air pressure changes from Po to Pr too fast, so there's not enough time to transfer the heat from the air to the container, the process is called adiabatic expansion. The change of state refers to the diagram 1. And it will satisfy the above equation, we obtain,

$$P_0 V_0^{\gamma} = P_r V_1^{\gamma} \tag{10}$$

Wait for 5 minutes, the temperature of the container will return to the room temperature *Tr* , the pressure rises from *Pr* to *P2.* When a system adjusts its temperature to the room temperature called isothermal process. Because of the small volume of the U-shaped tube, we do not consider the volume change of the air when rebound. Then we know that V1 ≡ V2, and then by the ideal gas equation, we obtain

$$P_0 V_0 = P_2 V_2 \equiv P_2 V_1 \tag{11}$$

Based on the above equation (9) and (10), we obtain

$$\gamma = \frac{\ln(P_0/P_r)}{\ln(P_0/P_2)} \tag{12}$$

Then, the atmospheric pressure can be expressed as the fluid column height in the U-tube manometer,

$$\begin{aligned} P_0 &= h_0 + h_i \\ P_r &= h_0 \\ P_2 &= h_0 + h_f \end{aligned}$$

Substitute the above equations into equation (12)

$$\gamma = \frac{\ln(1+{h_i}/{h_0})}{\ln(1+{h_i}/{h_0})+\ln(1+{h_f}/{h_0})} \quad (13)$$

$h_0 >> hi, hf$, so $\frac{h_i}{h_0} << 1, \frac{h_f}{h_0} << 1$, then the above equation can be rewritten as

$$\gamma = \frac{h_i}{h_i - h_f} \quad (14)$$

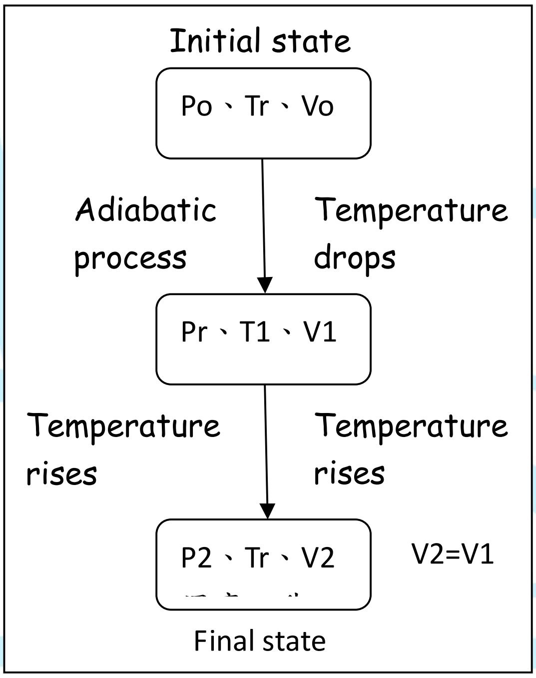


Diagram1- experimental state process

Instrument

NO	Accessory	Qty	NO	Accessory	Qty
1	Experimental Base	1	2	Scale Bracket	1
3	Removable Connector	1	4	Clamp	1
5	Valve	1	6	Flask	1
7	Silicone Tube	2	8	Air Ball	1
9	Transparent Tube	2			