Earth's Magnetic Field

Purpose

Students can learn how to use tangent galvanometers and understand constants and conversion constants of galvanometer. Compare the difference when an energized tangent galvanometer and an energized milliammeter are connected individually in series connection.

Theory

Suppose there is a short wire $d\vec{l}$ with current I. Based on the experimental result, the sine of angle θ between magnetic field $d\vec{B}$ and current I at point P of distance r (as shown in **Figure 1**), and wire length dl, $\vec{r}$ and $d\vec{l}$ is directly proportional and is inversely proportional to the square of distance r^2 so their relationship can be expressed as:

$$dB \propto \frac{Idl\sin\theta}{r^2} \qquad \therefore dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}\ ,\ \frac{\mu_0}{4\pi} = 10^{-7}\frac{N}{A^2} \tag{1}$$

In the formula, $\frac{\mu_0}{4\pi}$ is a constant of proportionality. μ_0 is the permeability of free space and its value is $4\pi\times10^{-7}\frac{N}{A^2}$. This formula is Boit-Savart law. If $\vec{e}_r$ is the unit vector on direction $\vec{r}$, the above vector formula can be written as:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{e}_r}{r^2} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times r\vec{e}_r}{r^3} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3} \tag{2}$$

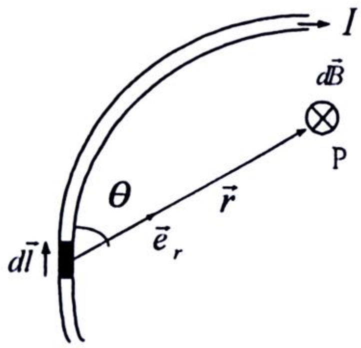

Figure 1 Figure of theoretical analysis

We can calculate the magnetic field at the centre of coil by formula (1) or (2).

Suppose a coil's radius is a and its current is I, as shown in **Figure 2**. The magnetic field $\vec{B}$ at center O of the coil can be calculated as below.

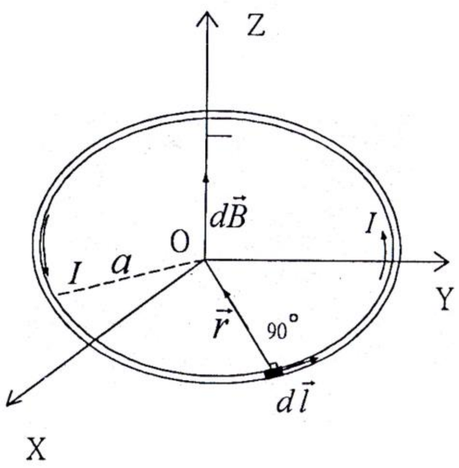

Figure 2 Magnetic field of a current coil

Calculate the magnetic field of short $d\vec{l}$ at coil to point O first. Because $d\vec{l} \perp \vec{r}$, we can then know the following relationship from Boit-Savart law:

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl}{a^2}$$

Do the integral to the coil. Because the directions of each section $d\vec{l}$ on the coil to the magnetic field of centre O are the same, the magnetic field at the centre of coil is

$$B = \frac{\mu_0}{4\pi}\frac{I}{a^2}\int_0^{2\pi a} dl = \frac{\mu_0}{4\pi}\frac{I}{a^2}\cdot 2\pi a = \frac{\mu_0}{2}\frac{I}{a} \qquad (3)$$

If the radius of tangent galvanometer is R and the laps of coil are N , according to formula (3), the magnetic field at the centre of tangent galvanometer is:

$$B = \frac{\mu_0}{2}\frac{NI}{R} \qquad (4)$$

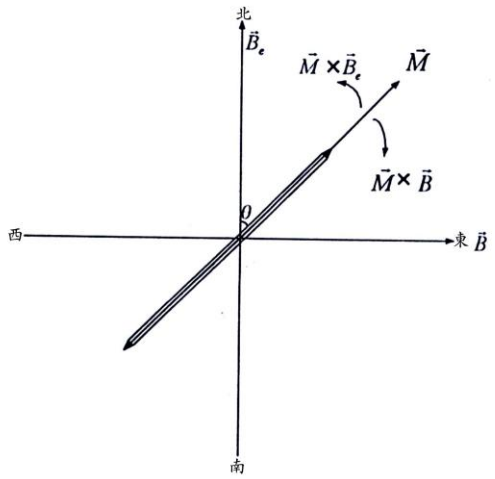

Figure 3 Torque of tangent galvanometer’s pointer

In **Figure 3**, the magnetic dipole moment of tangent galvanometer’s pointer is $\vec{M}$ (the pointing direction of N pole pointer). The direction of magnetic field $\vec{B}$ generated by coil is east. The horizontal component of Earth’s magnetic field is $\vec{B}_e$. According to $\vec{\tau} = \vec{M} \times \vec{B}$, if the angle between the North Pole pointer and the Earth’s magnetic field is θ, the torque of pointer generated by the magnetic field $\vec{B}$ is:

$$\tau = MB\sin(\frac{\pi}{2} - \theta) = MB\cos\theta \qquad \text{Clockwise}$$

The torque of pointer influenced by the Earth’s magnetic field is:

$$\tau_e = MB_e \sin\theta \qquad \text{Counter clockwise}$$

When the pointer of a tangent galvanometer reaches equilibrium, the above torques are equal but in opposite directions. When we put a magnetic pointer M at the center of vertical round coil C, the magnetic pointer is at equilibrium so:

$$\tau = MB\cos\theta = MB_e \sin\theta = \tau_e$$

$$\therefore \quad \tan\theta = \frac{B}{B_e} = \frac{\mu_0 NI}{2RB_e}$$

$$\Rightarrow \quad I = \frac{2RB_e}{N\mu_0}\tan\theta = \frac{B_e}{G}\tan\theta \qquad (5)$$

In formula (5), $G = {N\mu_0}/{2R}$ is the galvanometer constant which can be calculated by the coil's laps N and radius R. The conversion constant ${B_e}/{G}$ can be calculated by the horizontal intensity of the earth's magnetic B_e and G. If we make B_e / G =K, we can then acquire:

$$\Rightarrow \quad I = K\tan\theta \qquad \propto \tan\theta$$

Instruments

Instrument list					
No.	**Accessory**	**Qty**	**No.**	**Accessory**	**Qty**
1	Tangent galvanometer	1	2	Large goniometer	1
3	Current restricted power supply	1	4	Connecting wire	2

Instrument pictures

Introduction of instruments：

① Tangent galvanometer：

A tangent galvanometer is composed by a metal ring circulated by 40 laps of copper wires. The coil is placed on a vertical plane. There are three connectors, 0 laps, 15 laps and 40 laps, on the coil. If you want to use 15 laps, connect the connector between 0 and 15 laps. If you want to use 25 laps, connect the connector between 15 and 40 laps.

③ Current restricted power supply：

To prolong the life of instrument, please reset all the buttons before turning on the power. Reset all the buttons after use and the turn off the power.

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